

3E1456

Roll No. _____

[Total No. of Pages : **4**]**3E1456****CE****III****B.Tech. IIIrd Semester (Main/Back) Examination, Feb. - 2011****3CE6 Engineering Mathematics****Time : 3 Hours****Maximum Marks : 80****Min. Passing Marks : 24****Instructions to Candidates:**

Attempt five questions in all selecting one question from each unit. Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

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Unit - I

1. a) Find the Fourier series to represent $f(x) = x \cos x, -\pi < x < \pi$
- b) Obtain the first three cosine terms and the constant term in the Fourier series for y , where

$x:$	0	1	2	3	4	5
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$y:$	4	8	15	7	6	2
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OR

- a) Find the half range cosine series for the following function
 $f(x) = (x-1)^2, 0 < x < 1,$
 hence show that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

- b) The following table gives the variation of a periodic current over a period

$t(\text{secs})$: 0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (amps)	: 1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show by harmonic analysis that there is a direct current part of 0.75 amp. in the variable current and obtain the amplitude of the first harmonic.

Unit - II

2. a) If $L\{f(t)\} = \bar{f}(s)$, then prove that

$$L\{tf(t)\} = -\frac{d}{ds}\bar{f}(s)$$

Hence obtain $L\{te^a \cos bt\}$.

- b) Solve by the Laplace transform theory the equation

$$\frac{\partial^2 y}{\partial t^2} = 9 \frac{\partial^2 y}{\partial x^2}, \quad y(0, t) = 0, \quad y(2, t) = 0,$$

$$y(x, 0) = 20 \sin 2\pi x - 10 \sin 5\pi x \text{ and } y_t(x, 0) = 0.$$

OR

- a) Apply the convolution theorem to obtain $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$.

- b) Use Laplace transform theory to solve the differential equation:

$$(D^2 + 1)x = t \cos 2t, \quad x(0) = 0, \quad x'(0) = 0.$$

Unit - III

3. a) Find the Fourier Sine and cosine transform of $f(x) = \begin{cases} 1, & \text{for } 0 < x < a \\ 0, & \text{for } x > a \end{cases}$.

- b) Solve :

$$\frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial t^2},$$

given that $u_x(0, t) = 0$ and $u(x, 0) = \begin{cases} x, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$, $u(x, t)$ is bounded and $x > 0$, $t > 0$.

OR

- a) Find the Fourier transform of

$$f(x) = \begin{cases} x^2, & \text{when } |x| \leq a \\ 0, & \text{when } |x| > a \end{cases}$$

Hence evaluate

$$\int_0^\infty \cos \frac{as}{2} \left[(a^2 s^2 - 2) \sin as + 2as \cos as \right] \frac{ds}{s^3}.$$

- b) Using Fourier sine transform, solve the differential equation

$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x > 0$, $t > 0$ subject to the conditions :

$$u(0,t) = 0, \quad u(x,0) = \begin{cases} 1, & \text{when } 0 < x < 1 \\ 0, & \text{when } x \geq 1 \end{cases}$$

It may be assumed that $u(x,t)$ is bounded; also u and $\frac{\partial u}{\partial x}$ approach zero as $x \rightarrow \infty$

Unit -IV

4. a) Prove that

$$\text{i)} \quad u_0 + \frac{xu_1}{[1]} + \frac{x^2}{[2]} u_2 + \frac{x^3}{[3]} u_3 + \dots = e^x \left[u_0 + x\Delta u_0 + \frac{x^2}{[2]} \Delta^2 u_0 + \dots \right]$$

$$\text{ii)} \quad u_1 x + u_2 x^2 + u_3 x^3 + \dots = \frac{x}{1-x} u_1 + \left(\frac{x}{1-x} \right)^2 \Delta u_1 + \left(\frac{x}{1-x} \right)^3 \Delta^2 u_1 + \dots$$

- b) A slider in a machine moves along a fixed straight rod. Its distance x (cm) along the rod is given below for various values of time t (sec s).

$t:$	0	0.1	0.2	0.3	0.4	0.5	0.6
$x:$	30.28	31.43	32.98	33.54	33.97	33.48	32.13

Evaluate i) $\frac{dx}{dt}$ for $t=0.1$

and ii) $\frac{dx}{dt}$ for $t=0.3$

OR

- a) i) Evaluate $\int_{-1.6}^{-1} e^x dx$ by Simpson's one third rule with six intervals.

ii) Prove that $\nabla \Delta = \partial^2 = \Delta - \nabla$

- b) Use Stirling's formula to compute $U_{12.2}$ from the following data:

$x:$	10	11	12	13	14
$10^5 u_x:$	23967	28060	31788	35209	38368

Unit - V

- 5. a)** Employ Euler's method to solve :

$$\frac{dy}{dx} = \frac{y^2 - x}{y^2 + x}, \text{ given } y = 1, x = 0.$$

Find y for $x = 0.1, 0.2$ and 0.3 .

- b)** Use Milne's predictor - corrector method to find the solution of the differential equation

$$\frac{dy}{dx} = x - y^2$$

for next value of x , given that

$$y(0) = 0.0000, \quad y(0.2) = 0.0200,$$

$$y(0.4) = 0.0795, \quad y(0.6) = 0.1762,$$

OR

- a)** Use Picard's method to solve

$$\frac{dy}{dx} = 1 + xy, \text{ with } x_0 = 2, y_0 = 0 \text{ up to third Order of approximation.}$$

- b)** Using Runge-Kutta fourth order method, find the approximate value of y for $x = 0.2$ if $\frac{dy}{dx} = x + y^2$ given that $y = 1$ when $x = 0$, step size $h = .1$.
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