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B.Tech. II Semester (Main/Back) Examination, June/July - 2016
202 Engg. Mathematics - II

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

Instructions to Candidates:

*Attempt any **five** questions, selecting one question from **each unit**. All questions carry **equal** marks. (Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

Unit - I

1. a) A plane passes through a fixed point (a,b,c) and cut the axis in A,B,C. Show that the locus of the centre of the sphere OABC is **(8)**

    $$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

    b) Two spheres of radii r_1 and r_2 cut orthogonally, prove the radius of their common circle is **(8)**

    $$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

    OR

1. a) Define right circular cone. Find the equation of the right circular cone whose vertex is origin, axis is x - axis and semi vertical angle is α. **(2+6=8)**

    b) Define right circular cylinder. Find the equation of a right circular cylinder whose axis is

    $$\frac{x-2}{2} = \frac{y-1}{1} = \frac{z}{3}$$

    and which passes through (0,0,1) **(2+6=8)**

Unit - II

Find the rank of the following matrix by reducing it to the normal form :

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(8)

b) Test the consistency of the following system of equations and if possible solve it:

$$2x - 3y + 7z = 5$$
$$3x + y - 3z = 13$$
$$3x + 19y - 47z = 32$$

(8)

OR

2. a) Find the eigen values and eigen vectors of the following matrix :

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$

(8)

b) State cayley Hamilton Theorem, verify it for the matrix.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$ and find A^{-1}.

(2+6=8)

Unit - III

3. a) A particle moves on the curve $x = 2t^2, y = t^2 - 4t \ \ z = 3t - 5$, where t denote time. Find the components of velocity and acceleration at t = 1 in the direction of vector $\hat{i} - 3\hat{j} + 2\hat{k}$.

(8)

b) Prove that :

i) $\nabla^2(r^n) = n(n+2)r^{n-2}$, if $r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

ii) $Curl(\vec{a} \times \vec{r}) = 2\vec{a}$, if $\vec{a}$ is a constant vector.

(4+4=8)

2E2002   (2)

OR

3. a) If $\vec{a}$ and $\vec{b}$ are differentiable vector functions, then show that :

    i) $div(\vec{a} \times \vec{b}) = \vec{b}.curl\,\vec{a} - \vec{a}.\,curl\,\vec{b}$

    ii) $div\,curl\,\vec{a} = 0$ **(5+3=8)**

  b) Evaluate $\int_c \vec{F}.d\vec{r}$, where $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, curve c is rectangle in the xy - plane bounded by x = 0; x = a; y = 0; y = b.   **(8)**

Unit - IV

4. a) Evaluate $\iint_s \vec{F}.\hat{n}\,ds$ by using Gauss's divergence theorem for $\vec{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron x = y = z = 0, x + y + z = 1.   **(8)**

  b) State stoke's theorem. Verify Green's theorem in the plane for $\oint_c \left[(xy + y^2)dx + x^2 dy \right]$, where c is the closed curve of the region bounded by y = x² and y = x.   **(2+6=8)**

OR

4. a) Obtain the Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following :

    i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots\ldots = \frac{\pi^2}{6}$

    ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots\ldots\ldots = \frac{\pi^2}{12}$ **(5+3=8)**

  b) Express f(x) in a fourier series upto the second harmonic for the following data :

x :	0	π/3	2π/3	π	4π/3	5π/3	2π
f(x) :	1.98	2.15	2.77	-0.22	-0.31	1.43	1.98

    (8

Unit - V

4. a) Solve the following differential equation in series.

(8)

$$\frac{d^2y}{dx^2} + x^2y = 0$$

b) Solve :

(8)

$$x\left(y^2 - z^2\right)p - y(z^2 + x^2)q = z(x^2 + y^2)$$

OR

5. a) Solve :

(8)

$$x^2p^2 + y^2q^2 = z^2$$

b) Find a complete integral of

$$q = \left(z + px\right)^2$$

by using charpit's method.

(8)

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