

- (b) If the side and angles of a plane triangle ABC vary in such a way that its circumradius remains constant, then prove that :

$$\frac{\delta a}{\cos A} + \frac{\delta b}{\cos B} + \frac{\delta c}{\cos C} = 0$$

where, $\delta a, \delta b$ and δc are small increments in sides a, b and c respectively.

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- 4 (a) Find the maximum value of u , where

$$u = \sin x \sin y \sin(x+y)$$

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- (b) Find the Maxima and minima of $u = x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. Interpret the result geometrically.

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UNIT - III

- 5 (a) Find the length of the arc of the parabola $x^2 = 4ay$ from the vertex to an extremity of the latus rectum.

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- (b) Find the surface area of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x -axis.

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- 6 (a) Evaluate the following integral by changing to polar coordinates :

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} \, dx \, dy$$

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- (b) Show that :

$$B(m, n) = a^m b^n \int_0^{\infty} \frac{x^{m-1}}{(ax+b)^{m+n}} = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

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UNIT - IV

7 Solve :

(i) $x \sin(y/x) dy = [y \sin(y/x) - x] dx$

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(ii) $\frac{dy}{dx} = \left[\frac{x + 2y - 3}{2x + y - 3} \right]$

4

(iii) $(x^3 + xy^4) dx + 2y^3 dy = 0$

4

(iv) $(x^3 y^3 - xy) dx = dy$

4

8 Solve :

(i) $\frac{d^2 y}{dx^2} + a^2 y = \sec ax$

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(ii) $\frac{d^3 y}{dx^3} - \frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} = 1 + x^2$

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(iii) $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

6

UNIT - V

9 (a) Solve :

$$x^2 \frac{d^2 y}{dx^2} - (x^2 + 2x) \frac{dy}{dx} + (x + 2)y = x^3 e^x$$

8

(b) Solve :

$$\frac{d^2 y}{dx^2} + (\tan x - 3 \cos x) \frac{dy}{dx} + 2y \cos^2 x = \cos^4 x$$

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10 (a) Solve by the method of variation of parameters :

$$(1-x) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = (1-x)^2$$

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(b) Solve :

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left[x + \frac{1}{x} \right]$$

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