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B.Tech. I Semester (Main) Examination, Dec. - 2018

BSC

1FY2-01 Engineering Mathematics - I

Time : 3 Hours **Maximum Marks : 160**

Instructions to Candidates:

*Attempt all **ten** questions from **Part A**, any **five** questions out of seven from **Part B and any four** questions out of five from **Part C**. (Schematic diagrams must be shown wherever necessary). Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.*

Part - A

(Answer should be given up to **25** words only). **All** questions are **compulsory**.

(10×3=30)

1. What is the value of $\Gamma\left(-\frac{1}{2}\right)$.

2. Find the value of $\int_0^{\pi/2} \sin^6\theta \cos^7\theta \, d\theta$.

3. Find whether series $\sum \frac{n}{n+10}$ is convergent or not?

4. Give an example of two divergent series whose sum is convergent.

5. Find sum of Fourier series of f(x) at x = 2 where $f(x)=\begin{cases}0, 0\le x<1\\ 1, 1\le x<2\end{cases}$.

6. State Parseval's Theorem.

7. Give an example of two variable function whose both partial derivatives exist but limit does not exist at origin.

8. Find the directions in which $f(x,y)=\frac{x^2}{2}+\frac{y^2}{2}$ increases most rapidly at the point (1,1).

9. Suppose the force field $F=\nabla f$ is the gradient of the function $f(x,y,z)=-\frac{1}{(x^2+y^2+z^2)}$. Find the work done by F in moving an object along a smooth curve C joining (1,0,0) to (0,0,2) that does not pass through origin.

10. Find $\iint_s \vec{r}.\hat{n}\,dS$ where S is a closed surface enclosing volume V and $\vec{r}=xi+yj+zk$.

Part - B

(Analytical/Problem solving questions). **Attempt any five questions.**

(5×10=50)

1. Find volume of the solid generated by the revolution of the curve $x=a\cos^3\theta, y=a\sin^3\theta$ about the x-axis.

2. Find Taylor series expansion of $f(x) = \cos 5x^2$ about the point $x=\pi$.

3. Obtain half range sine series for $f(x) = e^x$, in $0<x<1$.

4. If resistors of R_1, R_2 and R_3 ohms are connected in parallel to make an R-ohm resistor, find the value of $\partial R/\partial R_2$ when $R_1 = 30$, $R_2 = 45$ and $R_3 = 90$ ohms.

5. The derivative of f(x,y) at $P_0(1,2)$ in the direction of i+j is $2\sqrt{2}$ and in the direction of -2j is -3. What is the derivative of f in the direction of -i-2j?

6. Find the area of the region R in the xy-plane enclosed by the circle $x^2+y^2=4$ above the line y = 1 and below the line $y=\sqrt{3}x$.

7. Find the centroid of the region in the first quadrant that is bounded above by the line y = x and below by the parabola $y = x^2$.

Part - C

(Descriptive/Analytical/Problem Solving/Design question). **Attempt any four questions.**

(4×20=80)

1. **Find the value of $\int_0^\infty \cos x^2 dx$.**

2. Discuss the convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}} x^n$.

3. Find Fourier series representation of $f(x) = \begin{cases} 0, -\pi \le x \le 0 \\ \sin x, 0 < x < \pi \end{cases}$ and prove that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots = \frac{1}{2}$$

4. The plane $x + y + z = 1$ cuts the cylinder $x^2+y^2=1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from origin.

5. Verify Stoke's theorem for the hemisphere $S : x^2 + y^2 + z^2 = 9, z \ge 0$, its bounding circle $C : x^2 + y^2 = 9, z = 0$, and the field $\vec{F} = yi - xj$.