1E3101

Roll No [redacted]

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# 1E3101

## B. Tech. I - Sem. (Main / Back) Exam., - 2023

## 1FY2 – 01 Engineering Mathematics - I

**Time: 3 Hours** **Maximum Marks: 70**

*Instructions to Candidates:*

***Attempt all ten questions from Part A, five questions out of seven questions from Part B and three questions out of five from Part C.***

*Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used /calculated must be stated clearly.*

*Use of following supporting material is permitted during examination. (Mentioned in form No. 205)*

1. NIL 2. NIL

## PART – A [10×2=20]

**(Answer should be given up to 25 words only)**

**All questions are compulsory**

Q.1 Find the limit of the sequence $\langle x_n \rangle$, where $x_n = \frac{5n-3}{7n+8}$.

Q.2 Write the power series expansion of logarithm function.

Q.3 Evaluate $a_n$ in the Fourier series of the function $f(x) = x + x^2, -\pi < x < \pi$.

Q.4 Define Cauchy's $(E - \delta)$ definition of continuity.

Q.5 Write Euler's theorem on homogeneous function.

Q.6 Evaluate: $\int_0^{\infty} x^6 e^{-2x} dx$ by using beta – gamma function.

Q.7 Evaluate: $\iint xy\,dx\,dy$, where the region of integration is $x + y < 1$ in the positive quadrant.

Q.8 Change the order of integration of the following double integration:

$$\int_0^4 \int_x^{2\sqrt{x}} f(x, y)dxdy$$

Q.9 If $\vec{f} = x^2y\hat{i} - 2xy^2z\,\hat{j} + 3x^2z\hat{k}$, find div f at the point (3, –1, –2).

Q.10 State Stokes theorem.

[5×4=20]

## PART – B

### (Analytical/Problem solving questions)

### Attempt any five questions

Q.1 Prove that –

$$B(m, n) = \frac{\lceil m \lceil n}{\lceil (m+n)}$$

Q.2 Test the convergence of the following series –

$$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \text{-------}$$

Q.3 Find a Fourier series for the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and deduce the following: $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

Q.4 Find the equations of the tangent plane and normal to the surface –

$x^3 + y^3 + 3xyz = 3$ at the point (1, 2, –1).

Q.5 Evaluate the point where the function –

$x^3y^2(1 - x - y)$

Will have maxima. Also find the maximum value.

Q.6 Evaluate the integral –

$$\int_0^1 \int_0^x \frac{x^3dxdy}{\sqrt{x^2+y^2}}$$

by changing into polar coordinates.

Q.7 If $\vec{a}$ and $\vec{b}$ are differentiable vector point functions, then prove that –

$$\text{div}\left(\vec{a} + \vec{b}\right) = \vec{b}.\text{curl}\,\vec{a} - \vec{a}.\text{curl}\,\vec{b}$$

## PART – C

[3×10=30]

**(Descriptive/Analytical/Problem Solving/Design Questions)**

**Attempt any three questions**

Q.1 Find the volume of spindle shaped solid generated by revolving the Astroid about the x – axis –

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Q.2 If $u = \log x^3 + y^3 + z^3 - 3xyz$, then prove that –

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

Q.3 Find half range cosine series for the function –

$$f(x) = 2x - 1, 0 < x < 1$$

hence deduce that –

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{-------}$$

Q.4 Find the volume of the tetrahedron bounded by the co – ordinate planes and the plane –

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Q.5 State Gauss's divergence theorem. Verify Gauss's divergence theorem for $\vec{F} = xy\,\hat{i} + z^2\hat{j} + 2yz\hat{k}$ on the tetrahedron $x = y = z = 0$ and $x + y + z = 1$.

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