1E2002

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B. Tech. I - Sem. (Back) Exam., Dec. 2019

Common to all Branch

102(O) Engineering Mathematics-I

Time: 3 Hours **Maximum Marks: 80**

Min. Passing Marks: 26

Instructions to Candidates:

*Attempt any **five questions,** selecting **one question** from **each unit.** All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

UNIT-I

Q.1 (a) Find the asymptotes of the following Curve – [8]

$x^3 - 5x^2y + 8xy^2 - 4y^3 + x^2 - 3xy + 2y^2 - 1 = 0$

(b) Prove that for Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \rho = \frac{a^2b^2}{P}$, P being the perpendicular from the center upon the tangent at any point (x, y) [8]

OR

Q.1 (a) Show that the point of inflexion of the curve $y^2 = (x - a)^2 (x - b)$ lie on the straight line $3x + a = 4b$. [8]

(b) Trace the curve $y^2 (a + x) = x^2 (a - x)$ [8]

UNIT- II

Q.2 (a) If $x^x y^y z^z = c$ then prove that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = \frac{-1}{x \log ex}$ [8]

(b) If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}\tan u$ [8]

OR

Q.2 (a) Find the maximum value of – [8]

$u = \sin x \sin y \sin(x + y)$

(b) Find the maximum or minimum value of $u = x^2 + y^2 + z^2$ when $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$ [8]

UNIT- III

Q.3 (a) Find the surface area of the solid generated by the revolution of the astroid $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis. [8]

(b) Evaluate the Integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ changing into polar co – ordinates. [8]

OR

Q.3 (a) Evaluate $\int_0^1 \int_{e^x}^e \frac{1}{\log y} dxdy$ by changing the order of integration. [8]

(b) To prove Relation between Beta and Gamma Function [8]

$B(m, n) = \frac{\sqrt{m}\sqrt{n}}{\sqrt{m+n}}$ $(m > 0, n > 0)$

UNIT- IV

Q.4 Solve the following differential equations –

(a) $(1 + y^2)\, dx = (\tan^{-1} y - x)\, dy$ [5]

(b) $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$ [5]

(c) $(x^3 + xy^2 + a^2y)\, dx + (y^3 + yx^2 - a^2x)\, dy = 0$ [6]

OR

Q.4 Solve the following differential equations –

(a) $(D^2 + a^2)\, y = \sec ax$ [5]

(b) $(D^2 - 4D + 4)\, y = e^{2x} + \sin 2x$ [5]

(c) $(D^2 + 2D + 1)\, y = x \cos x$ [6]

UNIT- V

Q.5 (a) Solve – [8]

$$x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$

(b) Solve $\sin^2 x \frac{d^2y}{dx^2} = 2y$ given $y = \cot x$ is a solution. [8]

OR

Q.5 Solve

(a) $\frac{d^2y}{dx^2} + (3 \sin x - \cot x) \frac{dy}{dx} + (2 \sin^2 x)\, y = e^{-\cos x} \sin^2 x$ [8]

(b) Use the method of variation of parameters to solve – [8]

$$\frac{d^2y}{dx^2} - y = \frac{2}{1 + e^x}$$
