1E2002

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1E2002

B. Tech. I Sem. (Back) Exam., Dec. - 2017

102 (O) Engineering Mathematics-I

Time: 3 Hours **Maximum Marks: 80**

Min. Passing Marks: 26

Instructions to Candidates:

*Attempt any **five questions**, selecting **one question** from **each unit**. All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

UNIT-I

Q.1 (a) Find the asymptotes of the following curve: [8]

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$$

(b) Prove that the radius of curvature at any point (x, y) on the Astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is three times the length of the perpendicular from the origin on the tangent at that point. [8]

OR

Q.1 (a) Find the points of inflexion for the following curve: [8]

$$y(a^2 + x^2) = x^3$$

(b) Trace the curve: [8]

$x^3 + y^3 = 3axy$

UNIT-II

Q.2 (a) If $u = f(r)$, where $r^2 = x^2 + y^2$ then prove that: [8]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

(b) If $u = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ [8]

Then by using Euler's theorem prove that:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

OR

Q.2 (a) Find the percentage error in the area of an ellipse when an error of 1% is made in measuring its major and minor axis. [8]

(b) Find the extreme points and their nature for the function. [8]

$u = x^3 + y^3 - 3axy$

UNIT-III

Q.3 (a) Find the volume of the solid formed by the revolution of the loop of the curve: [8]

$y^2 (a + x) = x^2(a - x)$

about the x – axis.

(b) Evaluate: [8]

$\iint_A y\,dx\,dy$,

Where A is the region of integration bounded by the parabolas:

$y^2 = 4ax$ and $x^2 = 4ay$

OR

Q.3 (a) Evaluate the following integral by changing the order of integration: [8]

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}}\,dx\,dy$$

(b) Using Beta and Gamma function theory, prove that: [8]

$$B(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

UNIT-IV

Q.4 Solve the following differential equations: [5+5+6=16]

(a) $x\,dy - y\,(1 + xy)\,dx = 0$

(b) $(1 + xy)\,x\,dy + (1 - xy)\,y\,dx = 0$

(c) $(y^3 - 2x^2y)\,dx + (2xy^2 - x^3)\,dy = 0$

OR

Q.4 Solve the following differential equations: [5+5+6=16]

(a) $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx} + 4\frac{dy}{dx} - 8y = 0$

(b) $(D^2 - 2D + 1)\,y = x^2e^{3x}$

(c) $(D^3 + 2D^2 + D)\,y = e^{2x} + x^2 + x$

UNIT-V

Q.5 (a) Solve the following differential equation:

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$$ [8]

(b) Solve the following differential equation: [8]

$$(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$$

OR

Q.5 (a) Solve the following differential equation:

$$x^6\frac{d^2y}{dx^2}+3x^5\frac{dy}{dx}+a^2y=\frac{1}{x^2}$$ [8]

(b) Apply the method of variation of parameters to solve the following differential equation:

$$(1-x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=(1-x)^2$$ [8]
