Roll No. ______________ [Total No. of Pages : 3]

1E2002

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B.Tech. I Semester (Main/Back) Examination - 2015
102 Engg. Mathematics - I

Time : 3 Hours **Maximum Marks : 80**
Min. Passing Marks(Old Back) : 24
Min Passing Marks (Main/Back) : 26

Instructions to Candidates:

*Attempt any **five questions**, selecting **one question** from **each unit**. All questions carry **equal** marks.(Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.)*

Unit - I

1. a) Find the asymptotes of the following curve :
$x^3 - 5x^2y + 8xy^2 - 4y^3 + x^2 + 2y^2 - 3xy - 1 = 0$ **(8)**

b) Show that in the parabola $y^2 = 4ax$, the radius of curvature at any point p is $\frac{2(sp)^{3/2}}{\sqrt{a}}$, where s is the focus of the parabola. Also, if p_1, p_2 are radii of curvatures at the extremities of a focal chord of the above parabola, then $(p_1)^{-2/3} + (p_2)^{-2/3} = (2a)^{-2/3}$ **(8)**

OR

1. a) Trace the curve : $y^2(a+x) = x^2(a-x)$ **(8)**

b) Trace the curve : $r = a + b\cos\theta,\ a > b$ **(8)**

Unit - II

2. a) If $u = \sin^{-1}\left(\frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}\right)$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{20}\tan u$. **(8)**

b) ABC is an acute-angled triangle with fixed base BC. If $\delta b, \delta c, \delta A$ and δB are small increments in b,c,A and B respectively when the vertex A is given a small displacement δx parallel to BC. prove that $\delta A = h\delta x\left(\frac{1}{c^2} - \frac{1}{b^2}\right)$,where h is the unaltered height of the triangle. **(8)**

OR

2. a) If $u = f(x,y), x = r\cos\theta, y = r\sin\theta$ Show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$ **(8)**

b) Find points on the surface $z^2 = xy + 1$ whose distances from the origin are minimum. **(8)**

Unit - III

3. a) Prove that the surface and volume of the solid generated by the revolution of the tractrix $x = a\cos t + \frac{1}{2}a\log\tan^2\left(\frac{t}{2}\right), y = a\sin t$ about its asymptote are respectively equal to the surface and half the volume of a sphere of radius a. **(10)**

b) Find by double integration the area of the region enclosed by the following curves : $y^2 = 4ax$ and $x^2 = 4ay$ **(6)**

OR

3. a) Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{(x^2+y^2)}}$ by changing the order of integration. **(8)**

b) i) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}\,\left|\overline{\frac{1}{n}}\right.}{n\left|\overline{\left(\frac{1}{n}+\frac{1}{2}\right)}\right.}$ **(4)**

ii) Show that $B(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy$ **(4)**

Unit-IV

4. Solve the following differential equations :

a) $(1+y^2)dx = \left(\tan^{-1} y - x\right)dy$ **(5)**

b) $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$ **(6)**

c) $(x-y)^2\frac{dy}{dx} = a^2$ **(5)**

OR

4. Solve the following differential equations :

a) $(D^4 + 2D^3 - 3D^2)y = 3e^{2x} + 4\sin x$ **(5)**

b) $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$ **(5)**

c) $\dfrac{d^3y}{dx^3} + 2\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = e^{2x} + x^2 + x$ **(6)**

Unit-V

5. a) Solve :

$$x^3 \frac{d^2y}{dx^2} + 3x^2 \frac{dy}{dx} + xy = \sin(\log x)$$ **(8)**

b) Solve :

$$(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x+3)\frac{dy}{dx} + 2y = (x+1)e^x$$ **(8)**

OR

5. a) Solve :

$$Cosx\frac{d^2y}{dx^2} + \sin x\frac{dy}{dx} - 2y\cos^3 x = 2\cos^5 x$$ **(8)**

b) Solve by the method of variation of parameters :

$$(x+2)\frac{d^2y}{dx^2} - (2x+5)\frac{dy}{dx} + 2y = (x+1)e^x$$ **(8)**