1E2201

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B. Tech. I Sem. (Main) Exam., Dec. - 2017

MA-101 Engineering Mathematics-I

Time: 3 Hours

Maximum Marks: 80
Min. Passing Marks: 28

Instructions to Candidates:

*Attempt any **five questions,** including **Question No.1 which is Compulsory.** All questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.*

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

Q.1 Compulsory, Answer for each sub-question be given in about 25 words:

(a) Define concave upward and Concave downward. [2]

(b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that [2]

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(c) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$, where [2]

$u = e^x \sin y, \qquad v = x + \log \sin y$

(d) Change the order of integration only in [2]

$$\int_0^1 \int_{e^x}^{e} \frac{dy\, dx}{\log y}$$

(e) Find the area, by double integration, bounded by parabola $y^2 = 4ax$ and its latus rectum. [2]

(f) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point P (2, 1, 3) in the direction of the vector $\vec{a} = \hat{i} - 2\hat{k}$ [2]

(g) Prove that $\vec{F} = \left(y^2 \cos x + z^2\right)\hat{i} + (2y \sin x - 4)\hat{j} + \left(3xz^2 + 2\right)\hat{k}$ is a conservative field. [2]

(h) Write the Cartesian formula of Gauss divergence theorem. [2]

Q.2 (a) Find the asymptotes of the curve – [8]

$$4x^3 - x^2y - 4xy^2 + y^3 + 3x^2 + 2xy - y^2 - 7 = 0$$

(b) Transform the integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y^2 \sqrt{\left(x^2 + y^2\right)}\,dx\,dy$ by changing to polar coordinates, and, hence evaluate it. [8]

Q.3 (a) Trace the curve $x^3 + y^3 = 3axy$ [8]

(b) Evaluate $\iiint_V x^2\,dx\,dy\,dz$ over the region V enclosed by the planes [8]

$$x = 0, y = 0, z = 0 \text{ and } x + y + z = a$$

Q.4 (a) Let $f(x, y) = \dfrac{x^3 - y^3}{x^2 + y^2}$, when $(x, y) \neq (0,0)$ and f (0,0) = 0. Show that the function f is continuous but not differentiable at the origin. [8]

(b) Prove that $\int_0^2 \left(8 - x^3\right)^{-1/3} dx = \dfrac{2\pi}{3\sqrt{3}}$ [8]

Q.5 (a) If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ [8]

(b) Prove that div $\left(r^n \vec{r}\right) = (n + 3) r^n$ [8]

Q.6 (a) Use Taylor's theorem to expand sin xy in powers of $(x - 1)$ and $(y - \pi/2)$ up to second- degree terms. [8]

(b) Verify Green's theorem in the plane for $\int_c (xy + y^2)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. [8]

Q.7 (a) Use Lagrange's method of multipliers to find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid. [8]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(b) Verify stoke's theorem for the vector field $\vec{F} = (x^2 - y^2)\, i + 2xy\, j$, integrated around the rectangle $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$ and $y = b$. [8]
